Dynamic Aspects of Strong Pinning

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We determine the current–voltage characteristic of type II superconductors in the presence of strong pinning centers. Focusing on a small density of defects, we derive a generic form for the characteristic with a linear flux-flow branch shifted by the critical current (excess-current characteristic). The details near onset, a hysteretic jump (for $\kappa \gg 1$) or a smooth velocity turn-on ($\kappa \to 1$), depend on the Labusch parameter κ characterising the pinning centers. Pushing the single-pin analysis into the weak pinning domain, we reproduce the collective pinning results for the critical current.

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The defining property of a (type II) superconductor is its ability to carry electric current without dissipation. This superflow is destroyed when the magnetic induction B enters the material in the form of quantized flux lines or vortices [1]: driven by the current density j via the Lorentz force $F_{\mathrm{L}} = jB/c$, the finite velocity v of vortices generates a dissipating electric field $E = vB/c$ parallel to j [2]. It is the material defects immobilizing vortices which reestablish the superflow of current, eventually rendering the superconductor amenable to technological applications. An elementary distinction is made in the design and action of pinning defects: *strong* pins act individually and generate large (plastic) deformations and metastable vortex states, while *weak* defects are unable to pin vortices alone and thus act collectively. In this letter, we determine the generic force–velocity (or current–voltage) characteristic of vortices driven by a current j and subject to a small density n_p of strong pins.

Vortex pinning has originally been studied by Labusch [3] for strong pins (see also Ref. 4) and has later been extended to weak collective pinning by Larkin and Ovchinnikov [5]. While the latter has been profoundly studied [6, 7], the further development of strong pinning theory has been less dynamic, although some progress has been made over time [8–12]. Recently, the two regimes have been analyzed within a pinning diagram [13] delineating the origin of static critical forces F_c as a function of defect density n_p and strength f_p. Here, we go beyond the calculation of the static critical force F_c and determine the full force $F_{\mathrm{L}} (= jB/c)$ versus velocity $v (= cE/B)$ (or j–E) characteristic of a so-called 'hard' type II superconductor. We focus on the single-pin–single-vortex strong pinning regime, implying that defects are dilute and moderately strong, pinning only one vortex line at a time; furthermore, we concentrate on isotropic material and ignore effects of thermal fluctuations.

The calculation of critical forces for weak pins involves dimensional [5, 6] or perturbative [14, 15] estimates and is rather on a qualitative level. Calculations of the force–velocity characteristic focus either on the perturbative regime at high velocities [14, 15] or on the universal regime near depinning [16]. The situation is different for strong pinning: here, the critical force and the full dynamical response can be determined quantitatively, once the shape of the pinning potential is known. The force–velocity characteristic we find agrees well with numerous (even textbook [17, 18]) experimental results [19–21]: a nearly linear flux-flow curve shifted by the critical force F_c (excess-current characteristic), with a hysteretic jump in velocity at onset for strong pinning changing to a smooth rise on approaching the weak pinning domain. Quite remarkably, continuing our single-pin analysis into the weak pinning domain, we can find the usual weak collective pinning results for the critical force. Below, we derive the formalism leading us to the force–velocity characteristic, present the results for the average pinning force $\langle F_p\rangle(v)$ for a Lorentzian-shaped pin, derive the generic characteristic for the strong pinning case in the dilute-pin limit, and finish with a rederivation of the weak collective pinning results for the critical current from a study of single-defect pinning.

The velocity–force characteristic derives from the dynamical equation for vortex motion

$$\eta v = F_{\mathrm{L}}(j) - \langle F_p\rangle(v) \tag{1}$$

with the Bardeen-Stephen [2] viscosity $\eta \sim BH_{c2}/\rho_n c^2$

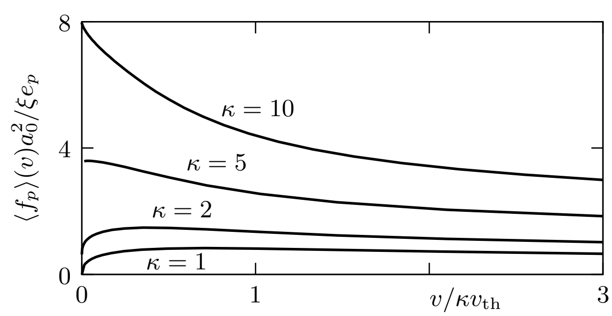

FIG. 1. Average pinning force $\langle f_p\rangle$ for a Lorentzian shaped pinning potential of various strengths. For strong pinning $\kappa \gg 1$ the critical force f_c is large and the pinning force decays monotonously. On approaching the Labusch point $\kappa \to 1$ the critical force f_c vanishes and the pinning force is non-monotonic, first increasing $\propto \sqrt{v}$ and then decaying $\propto 1/\sqrt{v}$.

(H_{c2} is the upper critical field and ρ_n denotes the normal state resistivity) and the velocity dependent average pinning force density $\langle F_p\rangle(v)$ (we choose $\langle F_p\rangle(v)$ to be positive). The pinning force density is determined by the positions $(\mathbf{R}_\mu + \mathbf{u}_\mu(z,t), z)$ of the flux lines (we choose $\mathbf{r}_\mu = (\mathbf{R}_\mu, z)$ to be the static lattice positions and $\mathbf{u}_\mu(z,t)$ is the vortex displacement field; the vortex density is $a_0^{-2} = B/\Phi_0$ with $\Phi_0 = hc/2e$ the flux quantum) and the individual pinning forces $\mathbf{f}_p(\mathbf{R}-\mathbf{R}_i)\delta(z-z_i)$ of defects located at positions $\mathbf{r}_i = (\mathbf{R}_i, z_i)$,

$$\langle \mathbf{F}_p\rangle = \frac{1}{N}\sum_\mu^N \int \frac{dz}{L}\,\mathbf{F}_\mathbf{p}(\mathbf{r}_\mu,\mathbf{u}_\mu), \quad \text{with} \tag{2}$$

$$\mathbf{F}_p(\mathbf{r}_\mu,\mathbf{u}_\mu) = \frac{-1}{a_0^2}\sum_i \mathbf{f}_p\big[\mathbf{R}_\mu + \mathbf{u}_\mu(z,t) - \mathbf{R}_i\big]\delta(z-z_i).$$

For a point-like defect $-e_p\delta(\mathbf{r})$, the convolution with the vortex shape $1-|\Psi(\mathbf{r})|^2 \approx 2\xi^2/(R^2+2\xi^2)$ (Ψ denotes the complex order parameter and ξ is the coherence length) provides the pinning potential

$$\varepsilon_p(\mathbf{R},z) = -e_p\frac{2\xi^2}{R^2+2\xi^2}\delta(z) \equiv e_p(\mathbf{R})\,\delta(z), \tag{3}$$

which here is of Lorentzian shape but may have another form in general. The pinning force is given by the gradient $\mathbf{f}_p(\mathbf{R}) = -\nabla_\mathbf{R} e_p(\mathbf{R})$.

The calculation of the pinning force density requires knowledge of the displacement field $\mathbf{u}_\mu(z,t)$. The latter is determined by the solution of the dynamical equation which we write in integral form

$$\mathbf{u}_\nu(z,t) = \mathbf{v}t + a_0^2\sum_\mu \int dz'dt'\,\hat{\mathbf{G}}(\mathbf{R}_\nu - \mathbf{R}_\mu, z-z', t-t') \times \mathbf{F}_p[\mathbf{r}'_\mu, \mathbf{u}_\mu(z',t')]. \tag{4}$$

The first term accounts for the Lorentz force in Eq. (1) generating the flux-flow velocity $v = F_\mathrm{L}/\eta$ in the absence of pinning. The dynamical elastic Green's function $\hat{\mathbf{G}}(\mathbf{r},t)$ is given by the Fourier transform of the matrix

$$G_{\alpha\beta}(\mathbf{k},\omega) = \frac{K_\alpha K_\beta/K^2}{c_{11}K^2 + c_{44}k_z^2 - i\eta\omega} + \frac{\delta_{\alpha\beta} - K_\alpha K_\beta/K^2}{c_{66}K^2 + c_{44}k_z^2 - i\eta\omega} \tag{5}$$

with the elastic moduli c_{11} (compression), c_{44} (tilt), and c_{66} (shear) [6]. The task simplifies considerably in the dilute-pin limit (to order n_p) and for moderately strong pinning defects trapping no more than one vortex; in this situation we can drop the sums over i and μ in Eqs. (2) and (4). We choose the pin position at the origin and let the vortex move on the x-axis; the problem then reduces to the calculation of the displacement field $u_x(z=0,t)$ at $z=0$. With the asymptotic position $x = vt$ (at $z=\pm\infty$), we have to solve the self-consistent equation

$$u(x) = x + \int_{-\infty}^{x}\frac{dx'}{v}\,G[0,(x-x')/v]\,f_p[u(x')], \tag{6}$$

where $u(x) = u_x(z=0,t)$ and with $G = G_{xx}$ and f_p the force along x. Inserting the solution back into Eq. (2), we obtain the pinning force density $F_p(0,z,u) = -f_p[u(vt)]\,\delta(z)$. The average pinning force density $n_p\,a_0^2\int dz\langle F_p\rangle$ due to a finite density n_p of defects involves the average $\langle\cdot\rangle$ over pin locations and time; the latter transform to an average along x and the impact parameter b of the vortex on the defect,

$$\langle F_p\rangle(v) = n_p\langle f_p\rangle = -n_p\Big\langle\int_{-\infty}^{\infty}\frac{dx}{a_\parallel}f_p[u(x)]\Big\rangle_b, \tag{7}$$

where $a_\parallel$ is the distance between vortices along the x direction. Restricting ourselves to the case of strongest pinning with $b=0$ and treating all trajectories within the range $\sigma\sim\xi$ of the pin equally, the average over impact parameters $\langle\cdot\rangle_b$ contributes a factor $\sigma/a_\perp$ with $a_\perp$ the transverse distance to the next vortex, hence $a_\parallel a_\perp = a_0^2$. Inserting the result for $\langle F_p\rangle(v)$ back into the dynamical equation (1) and solving for the velocity v for a given current j provides us with the desired result, the force–velocity characteristic of the superconductor.

In the static situation, the self-consistent integral equation (6) simplifies to the algebraic equation

$$u_s(x) = x + f_p[u_s(x)]/\bar{C}, \tag{8}$$

with $\bar{C}^{-1} = G(\mathbf{r}=0,\omega=0)$ the local static elastic Green's function, $\bar{C}\sim\varepsilon_0/a_0$ with $\varepsilon_0 = (\Phi_0/4\pi\lambda)^2$ the energy scale for vortices. Strong pinning is characterized by the appearance of bistable solutions in Eq. (8), implying that the derivative

$$\frac{d}{dx}\,f_p[u_s(x)] = -\bar{C}\frac{f'_p[u_s(x)]}{f'_p[u_s(x)]-\bar{C}} \tag{9}$$

of the effective force has to diverge—this provides us with the Labusch criterion [3] $\kappa \equiv \max_x\big\{f'_p[u_s(x)]/\bar{C}\big\} = 1$ separating weak ($\kappa<1$) and strong ($\kappa>1$) pinning. Note that the effective force gradient inside a very strong pin is universally given by the effective elastic constant $\bar{C}$, not by f'_p. The different solutions of Eq. (8) at $\kappa>1$ are associated with the unpinned (u_s outside the pin) and pinned (u_s inside the pin) states of the vortex; their asymmetric statistical occupation at finite drive produces a finite critical force density $F_c = \max\langle F_p\rangle(v=0)$ where the maximum is taken over the pinned and unpinned branches. For a weak pin ($\kappa<1$), Eq. (8) has a unique solution and the critical force density F_c vanishes.

In the dynamical situation with $v>0$ we have to solve the self-consistent integral equation (6) and thus need to know the time dependence of the Green's function $G(0,t)$. At short times $t<t_\mathrm{th} = \eta a_0^2/4\pi c_{66}$ the Green's function is dominated by the response of an individual vortex line (the one-dimensional (1D) regime), $G^{\mathrm{1D}}(0,t>0)\sim(t_\mathrm{th}/t)^{1/2}/\bar{C}t_\mathrm{th}$; at long times $t>t_\mathrm{th}\lambda^2/a_0^2$ the full 3D vortex system provides the response and $G^{\mathrm{3D}}(0,t>$

$0) \sim (t_{\rm th}/t)^{3/2} a_0/\bar{C} t_{\rm th} \lambda$ (the intermediate dispersive or 4D regime with $G^{\rm 4D} \sim (t_{\rm th}/t)^2/\bar{C} t_{\rm th}$ is less relevant in our analysis below).

At high velocities v the time integral in Eq. (6) extends over short times and the velocity-dependent part of the pinning force scales as $tG^{\rm 1D} \propto \sqrt{t}$, while, at small velocities, long times are relevant and $tG^{\rm 3D} \propto \sqrt{1/t}$. The time t to velocity v transformation $t \sim \sigma_{\rm eff}/v$ involves the effective pin size $\sigma_{\rm eff} \sim \kappa\xi/(1 + v/\kappa v_{\rm th})$ which depends on the pinning strength κ and on the velocity v itself [22]; for $\kappa \to 1$ and at high velocities $v > \kappa^2 v_{\rm th}$ the effective pin size $\sigma_{\rm eff}$ saturates at the true geometric pin size ξ (here, $v_{\rm th} \sim \xi/t_{\rm th}$ is the basic velocity scale). The corrections to the critical force F_c at small velocities $v < (a_0^2/\lambda^2)\kappa v_{\rm th}$ then are expected to scale as $\sqrt{v/\kappa v_{\rm th}}$, while the high-velocity $v > \kappa^2 v_{\rm th}$ corrections to the dissipative force ηv (flux-flow) decay as $\sqrt{v_{\rm th}/v}$. This is confirmed by the numerical solution of the problem following the steps indicated above and where the results are shown in Fig. 1 (we assume non-dispersive moduli corresponding to a field $B \sim \Phi_0/\lambda^2$). The forward integration of Eq. (6) has been done for a Lorentzian-shaped potential of the form (3) and different pinning strengths as expressed by the Labusch parameter $\kappa \sim (e_p/\xi\varepsilon_0)(a_0/\xi)$; with $e_p \sim H_c^2\xi^3 \sim \varepsilon_0\xi$ (H_c the thermodynamic critical field) the Labusch parameter can naturally access large numbers $\kappa \sim a_0/\xi \gg 1$. The scaled average pinning force $\langle f_p\rangle a_0^2/e_p\xi$ is plotted against the scaled velocity $v/\kappa v_{\rm th}$ and exhibits a monotonic decrease at large κ and a non-monotonic behavior enforced by the vanishing of the critical force $f_c = \langle f_p\rangle(0)$ as $\kappa \to 1$. While our rough estimate above correctly predicts the shape $\propto \sqrt{v/\kappa v_{\rm th}}$ of the finite-velocity corrections, its sign depends on κ in a nontrivial way [22]. Note that we plot the single-pin result $\langle f_p\rangle$ rather than the corresponding force density $\langle F_p\rangle$ as the density n_p is an important independent parameter.

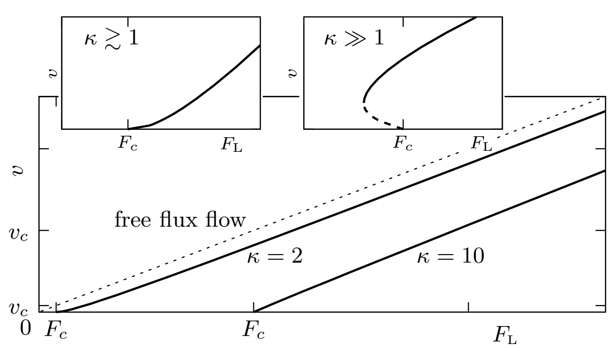

FIG. 2. Force-velocity characteristic for a Lorentzian-shaped pinning potential. For small defect densities $n_p a_0 \xi^2 \kappa = 0.05$ we find an excess-current characteristic, a shifted (by $F_c \propto n_p$) linear curve with a slope reflecting flux-flow behavior and approaching the true (unshifted) flux-flow behavior only at high velocities $\kappa v_{\rm th} \gg v_c$. The insets sketch the behavior near critical, with a hysteretic jump of order $v_{\rm nl} \propto n_p^2$ appearing at strong pinning $\kappa \gg 1$ and a smooth onset $v \propto v_{\rm th}(\kappa - 1)^4\,(F_{\rm L}/F_c - 1)^2$ on approaching the Labusch point $\kappa \to 1$.

In our discussion of the force–velocity characteristic we first concentrate on the overall shape away from the onset of vortex motion. The generic characteristic

$$\frac{F_{\rm L}}{F_c} = \frac{v}{v_c} + \frac{\langle f_p\rangle(v/\kappa v_{\rm th})}{f_c} \tag{10}$$

involves two velocity scales, the velocity $\kappa v_{\rm th}$ governing the pinning force $\langle f_p\rangle$ (as confirmed by a detailed analysis of Eq. (7) [22]) and the scale $v_c = F_c/\eta$ appearing from the competition between the dissipative (ηv) and the critical ($F_c = n_p f_c$) force densities. In the limit of small pin densities n_p, the linear term in Eq. (10) changes on the small velocity scale $v_c \propto n_p$, while the pinning force $\langle f_p\rangle(v)$ deviates from its static value f_c only on the larger scale $\kappa v_{\rm th}$ which does not depend on n_p. Indeed, making use of the expression $F_c \sim (\xi^2/a_0^2) n_p f_p (\kappa-1)^2/\kappa$ for the critical force density [13], we find the ratio $v_c/\kappa v_{\rm th} \sim n_p a_0 \xi^2 (\kappa-1)^2/\kappa \ll 1$ in the small pin density limit and at fixed κ (note that the limit $\kappa \to 1$ at fixed n_p would take us out of the single-pin regime). With $\langle f_p\rangle(v) \approx f_c$ for velocities $v \sim v_c \ll \kappa v_{\rm th}$ we find a characteristic that takes the generic form of a shifted (by F_c) linear (flux-flow) curve, $v \approx (F_{\rm L} - F_c)/\eta$, see Fig. 2; the free dissipative flow $v = F_{\rm L}/\eta$ is approached only at very high velocities $v \gg \kappa v_{\rm th} \gg v_c$. The simple excess-current characteristic is a consequence of the separation of velocity scales v_c and $\kappa v_{\rm th}$; the latter merge at strong pinning with increasing density n_p when strong 3D pinning goes over into 1D strong pinning at $n_p a_0 \xi^2 \kappa \sim 1$ [13]. Using qualitative arguments, a similar excess current characteristic has been found in Ref. 8.

The above simple overall structure of the force–velocity characteristic is modified at very small velocities and in close vicinity to the critical force density F_c; in this regime we can rewrite Eq. (10) in the simple form $F_{\rm L}/F_c = v/v_c + 1 \pm (v/v_p^\pm)^{1/2}$, where the '+' ('−') sign applies to the limits $\kappa \to 1$ ($\kappa \gg 1$). The small-velocity pinning scales $v_p^\pm$ derive from the 3D expression of the pinning force density [22] $\langle F_p\rangle - F_c \sim (\xi^2/a_0\lambda) n_p f_p \kappa \sqrt{v/\kappa v_{\rm th}}$ (at large κ), $v_p^- \sim (\lambda^2/a_0^2)\kappa\, v_{\rm th}$ for $\kappa \gg 1$ and $v_p^+ \sim (\lambda^2/a_0^2)(\kappa-1)^4\, v_{\rm th}$ for $\kappa \to 1$. For strong pinning, the negative (non-linear) correction in the average pinning force density generates a bistability (and hence hysteretic jumps) on the scale $v_{\rm nl} \sim v_c^2/v_p^- \propto n_p^2$. On the other hand, approaching the Labusch point, the correction changes sign and the velocity increases quadratically $v \sim v_p^+(F_{\rm L}/F_c - 1)^2$ until crossing over into the linear regime at $v_{\rm nl} \sim v_c^2/v_p^+ \ll v_c \ll v_p^+$. These features are visible in the insets of Fig. 2 showing an expanded view of the characteristic near onset.

Next, we push our single-pin (SP) analysis into the weak pinning domain $\kappa < 1$ and establish its relation to weak collective pinning (WCP) theory. In the dynamical formulation of WCP, we determine the pinning

force $\langle F_p\rangle^{\rm WCP}(v)$ perturbatively (to lowest order in κ and n_p) at high velocities and follow the velocity correction $\delta v = \langle F_p\rangle^{\rm WCP}(v)/\eta$ down to small v. As the correction δv becomes of order v, higher order terms become relevant [15] and we stop the analysis, interpreting the breakdown of perturbation theory as the signature of a finite critical force $F_c^{\rm WCP}$. The latter then derives from the critical velocity v_c defined through the criterion $\langle F_p\rangle^{\rm WCP}(v_c) \sim \eta v_c = F_c^{\rm WCP}$.

Within the SP analysis valid at small densities n_p, we usually calculate the pinning force density $\langle F_p\rangle^{\rm SP}(v)$ exactly, cf. Fig. 1; in the case of weak pinning $\kappa \ll 1$, we can use perturbation theory as well and we find the result

$$\langle F_p\rangle^{\rm SP}(v) \approx \int_0^\infty dt\, G(0,t)\, K^{x\alpha\alpha}(vt,0), \qquad (11)$$

where we have expanded the average pinning force density Eq. (7) for a displacement $u(x) = x + \delta u(x)$ close to flux-flow and used the lowest order (in κ) approximation of Eq. (6) for $\delta u(x)$. In Eq. (11), $K(\mathbf{u}) = (n_p/a_0^2)\int d^2R\, e_p(\mathbf{R}-\mathbf{u})\, e_p(\mathbf{R})$ replaces the usual pinning energy correlator showing up in WCP theory [6] (the superscripts denote derivatives with respect to u_x and u_α). Hence, the corrections δv from both the WCP- and the SP analysis agree with one another to lowest order in κ and in the pin density n_p. The difference in the two approaches arises when we take the velocity v to zero: While we stop at $\delta v \sim v$ and arrive at a finite $F_c^{\rm WCP}$ in WCP, we take v all the way to zero within the SP analysis and obtain a vanishing critical force $F_c^{\rm SP} = 0$. On the other hand, using the SP result Eq. (11) and adopting the WCP cutoff, we find a finite critical current as well: with the estimate $\langle F_p\rangle^{\rm SP}(v) \sim n_p(\xi/\lambda)(f_p^2/\varepsilon_0)(v/v_{\rm th})^{1/2}$ valid at low velocities and the conditions $\langle F_p\rangle^{\rm SP}(v_c) \sim \eta v_c \sim j_c B/c$, we obtain the critical current

$$j_c \sim j_0(\xi^2/\lambda)^2\big(n_p a_0^3 f_p^2/\epsilon_0^2\big)^2 \propto n_p^2, \qquad (12)$$

in agreement with the results obtained from weak collective pinning theory [13]. This result is quite remarkable: first, the critical current (12) is proportional to n_p^2, the *square* of the pin density n_p, *i.e.*, its origin is in the correlations between pins. Second, the result is still consistent with the standard SP result $\langle F_p\rangle^{\rm SP}(v=0) = 0$, as the latter is an order n_p result and corrections $\propto n_p^2$ are beyond the standard SP approach. Going back to strong pinning $\kappa > 1$, we already obtain a finite critical force $\langle F_p\rangle^{\rm SP}(v=0) \propto n_p$, *linear* in pin density. Pin-pin correlations then are expected to provide corrections $o(n_p)$ which vanish faster than linear and we can approach the critical force parametrically closer than in the WCP case.

Comparing our theoretical results to typical measured current–voltage characteristics, we find good agreement with experimental results [17–21]. The excess-current characteristic reported in these experiments was pointed out early on by Campbell and Evetts[4], however, we are not aware of any 'microscopic' derivation of this basic result. Unfortunately, a detailed comparison between theory and experiment is still not available today. Given a specific material, the defect structure is usually nontrivial and may include a variety of pin types. Furthermore, the parameters characterizing the defects are difficult to find. Experiments with superconductors where defects could be designed, tuned, and properly characterized would provide a great help and motivation in further developing the theory of pinning, particularly the crossover regime between strong and weak collective manifesting itself first in the small-velocity domain.

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